Definitions

Rosen (6th Edition) 2.1, 2.2

Basic Definitions

- Set - Collection of objects, usually denoted by a capital letter
- Member, element - Object in a set, usually denoted by a lower case letter
- Set Membership - \( c \in A \) denotes that \( c \) is an element of set \( A \)
- Cardinality of a set - Number of elements in a set, denoted \(|S|\)

Special Sets

- \( N \) - set of natural numbers = \{0,1,2,3,4, \ldots\}
- \( P \) or \( Z^+ \) - set of positive integers = \{1,2,3,4, \ldots\}
- \( Z \) - set of all integers, positive, negative and zero
- \( R \) - set of all real numbers
- \( \emptyset \) or \{\} - empty set
- \( U \) - Universal set, set containing all elements under consideration

Set Builder Notation

Format: "such that"

\{[element structure] : [necessary properties]}\n
Examples:

- \( Q = \{m/n : m,n \in Z, n\neq 0\} \)
  - \( Q \) is set of all rational numbers
  - Elements have structure \( m/n \), must satisfy properties after the colon.
- \( \{x \in R : x^2 = 1\} \)
  - \{-1,1\}

Subsets

- \( S \subseteq T \) (\( S \) is a subset of \( T \))
  - Every element of \( S \) is in \( T \)
  - \( \forall x (x \in S \rightarrow x \in T) \)
- \( S = T \) (\( S \) equals \( T \))
  - Exactly same elements in \( S \) and \( T \)
  - \( (S \subseteq T) \land (T \subseteq S) \) \text{ Important for proofs!} \n- \( S \subset T \) (\( S \) is a proper subset of \( T \))
  - \( S \) is a subset of \( T \) but \( S \neq T \)
  - \( (S \subseteq T) \land (S \neq T) \)

Examples

- \( \emptyset \subseteq S \forall S \)
- All subsets of \( S \)\( =\{a,b,c\} \)
  - \( \emptyset \)
  - \( \{a\}, \{b\}, \{c\} \)
  - \( \{a,b\}, \{b,c\}, \{a,c\} \)
  - \( \{a,b,c\} \)
- What set has the same quantity as both an element and a subset?
  - \( \{a, \{a\}\} \)

- Power Set \( P(S) \)
  - Set of all subsets of \( S \)
  - Cardinality of the power set is \( 2^n \) where \( n \) is \( |S| \)
  - If \( |S| = 3 \), then \( |P(S)| = 8 \)
Interval Notation - Special notation for subset of $\mathbb{R}$

- \([a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}\)
- \((a,b) = \{x \in \mathbb{R} : a < x < b\}\)
- \([a,b) = \{x \in \mathbb{R} : a \leq x < b\}\)
- \((a,b] = \{x \in \mathbb{R} : a < x \leq b\}\)

How many elements in \([0,1]\)?
In \((0,1)\)?
In \{0,1\}

Set Theory

Set Theory Operations

- $\bar{B}$ (B complement)
  - $\{x : x \in U \land x \notin B\}$
  - Everything in the Universal set that is not in $B$
- $A \cup B$ (A union B)
  - $\{x : x \in A \lor x \in B\}$
  - Like inclusive or, can be in $A$ or $B$ or both

More Set Operations

- $A \cap B$ (A intersect B)
  - $\{x : x \in A \land x \in B\}$
  - $A$ and $B$ are disjoint if $A \cap B = \emptyset$
- $A - B$ (A minus B or difference)
  - $\{x : x \in A \land x \notin B\}$
  - $A - B = A \cap \overline{B}$

More Set Operations

- $A \oplus B$ (symmetric difference)
  - $\{x : x \in A \oplus x \in B\} = (A \cup B) - (A \cap B)$
  - We have overloaded the symbol $\oplus$. Used in logic to mean exclusive or and in sets to mean symmetric difference

Simple Examples

Let $A = \{n^2 : n \in \mathbb{P} \land n \leq 4\} = \{1,9,16\}$
Let $B = \{n^4 : n \in \mathbb{P} \land n \leq 4\} = \{1,16,81,256\}$
- $A \cup B = \{1,4,9,16,81,256\}$
- $A \cap B = \{1,16\}$
- $A - B = \{4,9\}$
- $B - A = \{81, 256\}$
- $A \oplus B = \{4,9,81,256\}$
Determine whether each of these statements is true or false.

\[ x \in \{x\} \quad \text{T} \]
\[ \{x\} \subseteq \{x\} \quad \text{T} \]
\[ \{x\} \in \{x\} \quad \text{F} \]
\[ \{x\} \in \{\{x\}\} \quad \text{T} \]
\[ \phi \subseteq \{x\} \quad \text{T} \]
\[ \phi \in \{x\} \quad \text{F} \]
\[ \{x\} \subset \{x\} \quad \text{F} \]