An overview of Compression

Compressing becomes necessary in multimedia because it requires large amounts of storage space and bandwidth.

Types of Compression

- **Lossless compression**: data is not altered or lost in the process.
- **Lossy**: some info is lost but can be reasonably reproduced using the data.

Binary Image Compression

- **RLE (Run Length Encoding)**
  - Also called Packed Bits encoding
  - E.g. aaaaaaaaaaaaaaaaaaa111110000000
  - Can be coded as:
    - Byte1 Byte 2 Byte3 Byte4 Byte5 Byte6
    - 20 a 05 1 07 0
  - This is a one dimensional scheme. Some schemes will also use a flag to separate the data bytes.

Disadvantage of RLE scheme:

- When groups of adjacent pixels change rapidly, the run length will be shorter. It could take more bits for the code to represent the run length that the uncompressed data → negative compression.
- It is a generalization of zero suppression, which assumes that just one symbol appears particularly often in sequences.

Lossless Compression Algorithms (Entropy Encoding)

Adapted from:
http://www.cs.cf.ac.uk/Dave/Multimedia/node207.html

Basics of Information Theory

According to Shannon, the entropy of an information source $S$ is defined as:

$$H(S) = \eta = \sum_i p_i \log \frac{1}{p_i}$$

where $p_i$ is the probability that symbol $S_i$ in $S$ will occur.

$\log \frac{1}{p_i}$ indicates the amount of information contained in $S_i$, i.e., the number of bits needed to code $S_i$.

For example, in an image with uniform distribution of grey-level intensity, i.e., $p_i = 1/256$, then the number of bits needed to code each grey level is 8 bits. The entropy of this image is 8.
The Shannon-Fano Algorithm

A simple example will be used to illustrate the algorithm:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>15</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Encoding for the Shannon-Fano Algorithm:

- A top-down approach
- 1. Sort symbols according to their frequencies/probabilities, e.g., ABCDE.
- 2. Recursively divide into two parts, each with approx. same number of counts.

The Shannon-Fano Algorithm

Symbol | Count | log₂(1/p_i) | Code | Subtotal (# of bits)
--------|-------|-------------|------|-------------------|
A       | 15    | 1.38        | 00   | 30                |
B       | 7     | 2.48        | 01   | 14                |
C       | 6     | 2.70        | 10   | 12                |
D       | 6     | 2.70        | 110  | 18                |
E       | 5     | 2.96        | 111  | 15                |

TOTAL (# of bits): 89

Huffman Coding

Encoding for Huffman Algorithm:

- A bottom-up approach
- 1. Initialization: Put all nodes in an OPEN list, keep it sorted at all times (e.g., ABCDE).
- 2. Repeat until the OPEN list has only one node left:
  - (a) From OPEN pick two nodes having the lowest frequencies/probabilities, create a parent node of them.
  - (b) Assign the sum of the children’s frequencies/probabilities to the parent node and insert it into OPEN.
  - (c) Assign code 0, 1 to the two branches of the tree, and delete the children from OPEN.

Huffman Coding

| Symbol | Count | log₂(1/p_i) | Code | Subtotal (# of bits)
|--------|-------|-------------|------|-------------------|
A       | 15    | 1.38        | 0     | 15                |
B       | 7     | 2.48        | 100   | 21                |
C       | 6     | 2.70        | 101   | 18                |
D       | 6     | 2.70        | 110   | 18                |
E       | 5     | 2.96        | 111   | 15                |

TOTAL (# of bits): 87
Huffman Coding

Discussions:
- Decoding for the above two algorithms is trivial as long as the coding table (the statistics) is sent before the data. (There is a bit overhead for sending this, negligible if the data file is big.)
- Unique Prefix Property: no code is a prefix to any other code (all symbols are at the leaf nodes) → great for decoder, unambiguous.
- If prior statistics are available and accurate, then Huffman coding is very good.

In the above example:
entropy = \(\frac{15 \times 1.38 + 7 \times 2.48 + 6 \times 2.7 + 6 \times 2.7 + 5 \times 2.96}{39} = \frac{85.26}{39} = 2.19\)

Number of bits needed for Human Coding is: \(\frac{87}{39} = 2.23\)

Adaptive Huffman Coding

Motivations:
- (a) The previous algorithms require the statistical knowledge which is often not available (e.g., live audio, video).
- (b) Even when it is available, it could be a heavy overhead especially when many tables had to be sent when a non-order0 model is used, i.e. taking into account the impact of the previous symbol to the probability of the current symbol (e.g., "qu" often come together, ...).
- The solution is to use adaptive algorithms. As an example, the Adaptive Huffman Coding is examined below. The idea is however applicable to other adaptive compression algorithms.

Adaptive Huffman Coding

**ENCODER**

Initialize_model();
while ((c = getc (input)) != eof)
{
    encode (c, output);
    update_model (c);
}

Adaptive Huffman Coding

**DECODER**

Initialize_model();
while ((c = decode (input)) != eof)
{
    encode (c, output);
    update_model (c);
}

The key is to have both encoder and decoder to use exactly the same initialization and update_model routines.

**update_model** does two things: (a) increment the count, (b) update the Huffman tree.

During the updates, the Huffman tree will be maintained its sibling property, i.e. the nodes (internal and leaf) are arranged in order of increasing weights (see figure).

When swapping is necessary, the farthest node with weight \(W\) is swapped with the node whose weight has just been increased to \(W+1\).

Note: If the node with weight \(W\) has a sub tree beneath it, then the sub tree will go with it.

The Huffman tree could look very different after node swapping, e.g., in the third tree, node A is again swapped and becomes the #5 node. It is now encoded using only 2 bits.
Adaptive Huffman Coding

Note: Code for a particular symbol changes during the adaptive coding process.

Adaptive Huffman Tree

Increasing weight

Adaptive Huffman Coding

Lempel-Ziv-Welch Algorithm

Motivation:
- Suppose we want to encode the Webster's English dictionary which contains about 159,000 entries. Why not just transmit each word as an 18 bit number?
- Problems: (a) Too many bits, (b) everyone needs a dictionary, (c) only works for English text.
- Solution: Find a way to build the dictionary adaptively.
- Original methods due to Ziv and Lempel in 1977 and 1978. Terry Welch improved the scheme in 1984 (called LZW compression). It is used in e.g., UNIX compress, GIF, V.42 bis.

LZW Compression Algorithm

LZW Compression Algorithm:

Original LZW used dictionary with 4K entries, first 256 (0-255) are ASCII codes.

Example: Input string is **WED**^WE^**WEB**^WET**.

Steps:

<table>
<thead>
<tr>
<th>w</th>
<th>k</th>
<th>Output</th>
<th>Index</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>NIL</td>
<td>^</td>
<td></td>
<td>^</td>
<td></td>
</tr>
<tr>
<td>^</td>
<td>W</td>
<td>256</td>
<td>^W</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>E</td>
<td>257</td>
<td>WE</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>258</td>
<td>ED</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>^</td>
<td>259</td>
<td>D^</td>
<td></td>
</tr>
</tbody>
</table>
LZW Compression Algorithm

Steps:

1. Start with an empty dictionary.
2. Read a character k.
3. Output k.
4. Add k to the dictionary.
5. While (read a character k):
   a. If k is a new symbol, output entry for k.
   b. Add w + entry[0] to the dictionary.
   c. w = entry.

LZW Decompression Algorithm:

Example:

Input string is "^WED<256>E<260><261><257>B<260>T".

Steps:

<table>
<thead>
<tr>
<th>w</th>
<th>k</th>
<th>Output</th>
<th>Index</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>^</td>
<td>^</td>
<td>^</td>
<td>^</td>
<td>^</td>
</tr>
<tr>
<td>^W</td>
<td>W</td>
<td>256</td>
<td>^W</td>
<td>^W</td>
</tr>
<tr>
<td>W</td>
<td>E</td>
<td>257</td>
<td>WE</td>
<td>WE</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>258</td>
<td>ED</td>
<td>ED</td>
</tr>
<tr>
<td>D</td>
<td>&lt;256&gt;</td>
<td>259</td>
<td>^W</td>
<td>^W</td>
</tr>
</tbody>
</table>
**LZW Compression Algorithm**

**Steps:**

<table>
<thead>
<tr>
<th>Step</th>
<th>Symbol</th>
<th>Value</th>
<th>Code</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>&lt;256&gt;</td>
<td>E</td>
<td>E</td>
<td>WE</td>
</tr>
<tr>
<td>7</td>
<td>E</td>
<td>&lt;260&gt;</td>
<td>E</td>
<td>WE</td>
</tr>
<tr>
<td>8</td>
<td>&lt;260&gt;</td>
<td>&lt;261&gt;</td>
<td>E</td>
<td>WEE</td>
</tr>
<tr>
<td>9</td>
<td>&lt;261&gt;</td>
<td>&lt;257&gt;</td>
<td>WE</td>
<td>E^W</td>
</tr>
<tr>
<td>10</td>
<td>&lt;257&gt;</td>
<td>B</td>
<td>B</td>
<td>WEB</td>
</tr>
<tr>
<td>11</td>
<td>B</td>
<td>&lt;260&gt;</td>
<td>WE</td>
<td>B^</td>
</tr>
<tr>
<td>12</td>
<td>&lt;260&gt;</td>
<td>T</td>
<td>T</td>
<td>WET</td>
</tr>
</tbody>
</table>

**LZW Compression Algorithm**

**Problem:** What if we run out of dictionary space?

**Solution 1:** Keep track of unused entries and use LRU (Least Recently Used)

**Implementation Note:** LZW can be made really fast; it grabs a fixed number of bits from input stream, so bit parsing is very easy. Table lookup is automatic.

**Huffman vs. Arithmetic Code**

**Lowest \( L_{ave} \) for Huffman codes is 1. Suppose \( H < \frac{1}{2} \):**

- One option: use one code symbol for several source symbols
- Another option: Arithmetic code.

**Idea behind arithmetic code:**

Represent the probability of a sequence by a binary number.

**Arithmetic Encoding**

**Assume source alphabet has values 0 and 1, \( p_0 = p \), \( p_1 = 1 - p \).**

**A sequence of symbols \( s_1, s_2, \ldots, s_m \) is represented by a probability interval found as follows:**

- Initialize, \( lo = 0; hi = 1 \)
- For \( i = 0 \) to \( m \)
  - if \( s_i = 0 \)
    - \( hi = lo + (hi - lo) \cdot p_0 \)
  - else (\( s_i = 1 \))
    - \( lo = lo + (hi - lo) \cdot p_1 \)
- Send binary fraction \( x \) such that \( lo \leq x < hi \). This will require \( \lceil \log_2 (hi - lo) \rceil \) bits, where \( x = \sum_{i=1}^{m} \log_2 P(s_i) \)

**Arithmetic coding: example**

\( p_0 = 0.2 \), source sequence is 1101

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Low</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
<td>0.1280</td>
</tr>
<tr>
<td>1</td>
<td>0.1280</td>
<td>0.2560</td>
</tr>
<tr>
<td>2</td>
<td>0.2560</td>
<td>0.3840</td>
</tr>
<tr>
<td>3</td>
<td>0.3840</td>
<td>0.5120</td>
</tr>
</tbody>
</table>

Number of bits = \( \lceil \log_2(0.1024) \rceil = 4 \) Bits sent: 0111

\( p_0 = 0.2 \), source sequence is 1101

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Low</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2000</td>
<td>0.8000</td>
</tr>
<tr>
<td>1</td>
<td>0.8000</td>
<td>0.9600</td>
</tr>
<tr>
<td>2</td>
<td>0.9600</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.3856</td>
<td>0.1024</td>
</tr>
</tbody>
</table>

Number of bits = \( \lceil \log_2(0.1024) \rceil = 4 \) Bits sent: 0111
**Arithmetic Decoding**

We receive \( x \), a binary fraction

\[
\text{lo} = 0; \text{hi} = 1
\]

for \( i = 1 \) to \( m \)

- if \((x - \text{lo}) < p \cdot (\text{hi} - \text{lo})\)
  - \( s_i = 0 \)
  - \( \text{hi} = \text{lo} + (\text{hi} - \text{lo}) \cdot p \)
- else
  - \( s_i = 1 \)
  - \( \text{lo} = \text{lo} + (\text{hi} - \text{lo}) \cdot p \)

end

end

\( m \) and \( p \) are sent to the decoder in the header.

**Arithmetic Decoding**

We receive \( x \), a binary fraction

\[
\text{lo} = 0; range = 1;
\]

for \( i = 1 \) to \( m \)

- if \((x - \text{lo}) < p \cdot range\)
  - \( s_i = 0 \)
  - \( range = p \cdot range \)
- else
  - \( s_i = 1 \)
  - \( \text{lo} = \text{lo} + range \cdot p \)
  - \( range = range \cdot (1 - p) \)

end

end

**Arithmetic Decoding**

We receive \( x \), a binary fraction

for \( i = 1 \) to \( m \)

- if \( x < p \)
  - \( s_i = 0 \)
  - \( x = x/p \)
- Else if \( x > p \)
  - \( s_i = 1 \)
  - \( x = (x - p)/(1 - p) \)

end

end

**Magic Features of Arithmetic Coding**

Remember \( I \) (information) = \(- \log_2 p\)

- \( p = 0.5, I = 1 \)
- \( p = 0.125, I = 3 \)
- \( p = 0.99, I = 0.0145 \) (wow!)

High \( p \) symbol, less than 1 code bit per symbol!

In encoder, \( \text{hi} - \text{lo} = \sum I(\text{symbols}) \)

**Discussion on Arithmetic Coding**

When the length of the original can not be predetermined, how does the decoder know where to stop?

- A special ending symbol, similar to EOF.
- Sending fixed length chunks.

How do we determine the value for \( p \)?

Based on estimating the probabilities of symbols to appear in the sequence.

Can we encode data with symbols other than 0 and 1?
Conclusions

- Huffman maps fixed length symbols to variable length codes. Optimal only when symbol probabilities are powers of 2.
- Lempel-Ziv-Welch (LZW) is a dictionary-based compression method. It maps a variable number of symbols to a fixed length code.
- Adaptive algorithms do not need a priori estimation of probabilities, they are more useful in real applications.
- Arithmetic algorithms: Complexity: requires arithmetic (multiplications, divisions), rather than just table lookups
  - Algorithms are complex, accuracy (significant bits) is tricky
  - Can be made to operate incrementally
    - Both encoder and decoder can output symbols with limited internal memory
  - Provides important compression savings in certain settings

References

- Introduction to Data Compression, Khalid Sayood, Morgan Kaufmann, 1996.