IMPLICIT CROWDS: OPTIMIZATION INTEGRATOR FOR ROBUST CROWD SIMULATION

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Given desired velocities, how should agents navigate around each other?
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Adding Realism

- Probabilistic approaches [Wolinski et al. 2016]
- .....
LOCAL COLLISION AVOIDANCE

**Force-based methods** [Reynolds 1987, 1999; Helbing et al. 2000; Pelechano et al. 2007, …]

- Require very small time steps for stability

**Velocity-based methods** [van den Berg et al. 2008, 2011; Pettré et al. 2011, …]

- Overly conservative behavior

![Diagram](image1)

\[ \Delta t = 0.1 \text{s} \]
We seek a generic technique for multi-agent navigation that

- guarantees collision-free motion
- is robust to variations in scenario, density, time step
- exhibits high-fidelity behavior
- can update at footstep rates (0.3-0.5 s)
OUR CONTRIBUTIONS

1. General form of collision avoidance behaviors

\[
\frac{dv}{dt} = - \frac{\partial R(x, v)}{\partial v}
\]

supporting optimization-based implicit integration

2. Application to state-of-the-art power law model

\[ R(x, v) \propto \tau(x, v)^{-p} \]

for practical crowd simulations
I. OPTIMIZATION INTEGRATOR FOR CROWDS
IMPLICIT INTEGRATION

\[
\frac{d}{dt} \begin{bmatrix} X \\ v \end{bmatrix} = \begin{bmatrix} v \\ M^{-1} f(x, v) \end{bmatrix}
\]

• Unconditionally stable, but

[Baraff and Witkin, 1998]
IMPLICIT INTEGRATION

\[
\frac{d}{dt} \begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ M^{-1} f(x, v) \end{bmatrix} \iff
\]

\[
x^{n+1} - x^n = v^{n+1} \Delta t, \\
M(v^{n+1} - v^n) = f^{n+1} \Delta t
\]

• Unconditionally stable, but
• Need to solve a non linear system
• Slow (but we can use large time steps)

[Kaufman et al. 2014]
As long as forces are conservative:

\[ \mathbf{f}(\mathbf{x}) = -\frac{dU(\mathbf{x})}{d\mathbf{x}}, \]

we can express backward Euler in optimization form [Martin et al. 2011; Gast et al. 2015]:

\[ \mathbf{x}^{n+1} = \arg \min_{\mathbf{x}} \left( \frac{1}{2\Delta t^2} \| \mathbf{x} - \tilde{\mathbf{x}} \|_\mathbf{M}^2 + U(\mathbf{x}) \right) \]
As long as forces are *conservative*:

$$ f(x) = -\frac{dU(x)}{dx}, $$

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$$ x^{n+1} = \arg \min_x \left( \frac{1}{2\Delta t^2} \| x - \tilde{x} \|_M^2 + U(x) \right) $$

Interpretation: tradeoff between *maintaining velocity* and *reducing potential energy*
$x^{n+1} = \arg \min_x \left( \frac{1}{2\Delta t^2} \| x - \tilde{x} \|^2_M + U(x) \right)$

Why this is good:

- Simple and fast algorithms (e.g. gradient descent, Gauss-Seidel) can be given guarantees
- Highly nonlinear forces can be used without linearization
- Lots of recent advances in optimization for data mining, machine learning, image processing, …

[Fratarcangeli et al. 2016]
Conservative potentials $U(x)$ can only model \textit{position-dependent forces}
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- Humans anticipate
  - People anticipate future trajectories of others [Cutting et al. 2005, Olivier et al. 2012; Karamouzas et al. 2014]
  - Brains have special neurons for estimating collisions [Gabbiani 2002]

Crowd forces depend both on positions and velocities!
Hypothesis: Multi-agent interactions can be expressed as

$$f(x, v) = -\frac{\partial R(x, v)}{\partial v}$$

where $R$ is an anticipatory potential that drives agents away from high-cost velocities

(This is analogous to dissipation potentials [Goldstein 1980] in classical mechanics)
Optimization Integrator for Non-Conservative Forces

Anticipatory forces

\[ \mathbf{v}^{n+1} = \arg \min_{\mathbf{v}} \frac{1}{2} \| \mathbf{v} - \tilde{\mathbf{v}} \|_{M}^{2} + R(\mathbf{x} + \mathbf{v} \Delta t, \mathbf{v}) \Delta t \]
Optimization Integrator for Non-Conservative Forces

Anticipatory + conservative forces

\[ v^{n+1} = \arg \min_v \frac{1}{2} \|v - \hat{v}\|^2_\text{M} + U(x + v\Delta t) + R(x + v\Delta t, v)\Delta t \]

- First-order accurate
- \( U(\cdot) + R(\cdot)\Delta t \) is analogous to the “effective interaction potential” in symplectic integrators [Kane et al. 2000; Kharevych et al. 2006]
- Simple interpretation: tradeoff between maintaining velocity, reducing \( U \), and reducing \( R \)
SOME EXISTING MODELS

Alignment behavior in boids [Reynolds 1987]:

\[ f_{ij} = -w(\|x_{ij}\|)v_{ij} \iff R_{ij} = w(\|x_{ij}\|)\|v_{ij}\|^2 \]
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Velocity obstacles [Fiorini and Shiller 1998]:
\[ v_{ij} \notin VO(x_{ij}) \Leftrightarrow R_{ij} = \begin{cases} \infty & \text{if } v_{ij} \in VO(x_{ij}) \\ 0 & \text{otherwise} \end{cases} \]
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What is \( R_{ij} \) for humans?
II. IMPLICIT CROWDS USING THE POWER-LAW MODEL
For each pair of agents:
- Compute time to collision $\tau(x, v)$

Collisions occur when

$$\|x_{ij} + v_{ij}\tau\| = r_i + r_j$$

[Karamouzas et al. 2014]
For each pair of agents:

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$$
\|x_{ij} + v_{ij} \tau\| = r_i + r_j
$$
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- Compute potential $R(x, v) \propto \tau(x, v)^{-p}$

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[Karamouzas et al. 2014]
Problem: Apply power law potential to optimization-based backward Euler

Easy? Not quite…

- $R$ is discontinuous at boundary of collision cone
- $R$ becomes infinitely steep as agents graze past

Both phenomena cause numerical solvers to “get stuck”
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A CONTINUOUS TTC POTENTIAL

Discontinuity due to time to collision (finite if collision predicted, infinite if not)
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Solution:
Let’s work with $\frac{1}{\tau}$ (or, “imminence” of collision)

- Replace it with a continuous approximation, e.g., by linear extrapolation
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- Replace it with a continuous approximation, e.g., by linear extrapolation
- $R$ becomes $C^{p-1}$-smooth
Grazing trajectories make $R$ badly behaved.
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- Add some distance-based repulsion $U_{ij} \propto \frac{1}{\|x_{ij}\| - r_{ij}}$
- Continuous collision detection: replace distance $\|x_{ij}\|$ with *minimum* distance over the time step

Alternative approach to repulsion: add uncertainty to time-to-collision computation [Forootaninia et al. 2017]
III. ANALYSIS AND RESULTS
Implicit integration + continuous PowerLaw potential

• Guaranteed collision-free motion
• Smooth (C²-continuous) trajectories
Implicit integration + continuous PowerLaw potential

- Guaranteed collision-free motion
- Smooth (C^2-continuous) trajectories

Collision-free proof

- \( \mathbf{v}^{n+1} \) minimizes
  \[
  \frac{1}{2} \left\| \mathbf{v}^{n+1} - \mathbf{\tilde{v}} \right\|_M^2 + U(\mathbf{x}^{n+1}) + R(\mathbf{x}^{n+1}, \mathbf{v}^{n+1}) \Delta t
  \]

- \( R \) is infinite for a colliding state. \( U \) is infinite for a tunneling step. So these cannot be minima.
Comparisons to
- ORCA [van den Berg et al. 2011] (representative velocity-based approach)
- PowerLaw [Karamouzas et al. 2014] (non-continuous TTC + forward Euler)

<table>
<thead>
<tr>
<th>Scenario</th>
<th># Agents</th>
<th># Obstacles</th>
<th>Roadmap</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hallway</td>
<td>300</td>
<td>2</td>
<td>no</td>
<td>medium</td>
</tr>
<tr>
<td>Crossing</td>
<td>400</td>
<td>0</td>
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<tr>
<td>Evacuation</td>
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<td>178</td>
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</tr>
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<td>Blocks</td>
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COllision-Free Motion

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COLLISION-FREE MOTION

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Comparison to human crowds [Charalambous et al. 2014]
TIME STEP STABILITY

Implicit
\[ \Delta t = 0.4s \]

(1.5x playback speed)
TIME STEP STABILITY

Motion doesn’t change significantly with time step
PERFORMANCE

Cost is linear in number of agents, increases slowly with time step size
(Still, 2x-10x slower than ORCA or PowerLaw on a 6-core Intel Xeon E5-1650)

Future work: Improve performance via local-global alternating minimization techniques

[Liu et al. 2017]
[Narain et al. 2016]
LIMITATIONS AND FUTURE WORK

• Can other recent crowd models be formulated via interaction energies [Wolinski et al. 2016, Dutra et al. 2017; …]?
• Incorporating asymmetrical interactions, e.g., leader-following behavior
• What is the $\Delta t$ threshold where quality is maintained?

![Images showing crowd dynamics with different time intervals: $\Delta t=0.1s$, $\Delta t=1s$, $\Delta t=4s$]
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| $\Delta t$ | Simulated Crowd
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<tr>
<td>0.1s</td>
<td><img src="image1.png" alt="Simulation" /></td>
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<tr>
<td>1s</td>
<td><img src="image2.png" alt="Simulation" /></td>
</tr>
<tr>
<td>4s</td>
<td><img src="image3.png" alt="Simulation" /></td>
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FUTURE WORK

- Applications to LOD systems and footstep-based animation engines
- Applications to nonlinear dissipation forces in physics-based animation

[Zhu et al. 2015]

[Xu and Barbic 2017]
THANK YOU

https://www.cs.clemson.edu/~ioannis/implicit-crowds/