Regular Expressions

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A ***regular expression*** (RE) describes a language.

It uses the three ***regular*** operations. These are called ***union/or***, ***concatenation*** and ***star***.

Brackets (and) are used for grouping, just as in normal math.

Union

The symbol $+$ means ***union*** or ***or***.

Example:

$$0 + 1$$

means either a zero or a one.

Concatenation

The ***concatenation*** of two REs is obtained by writing the one after the other.

Example:

$$(\mathtt{0}+\mathtt{1})\,\mathtt{0}$$

corresponds to $\{\mathtt{00}, \mathtt{10}\}$.

$$(\mathtt{0}+\mathtt{1})\,(\mathtt{0}+\varepsilon)$$

corresponds to $\{\mathtt{00}, \mathtt{0}, \mathtt{10}, \mathtt{1}\}$.

Star

The symbol $*$ is pronounced star and means zero or more copies.

Example:

$$\mathtt{a}^*$$

corresponds to any string of $\mathtt{a}$'s: $\{\varepsilon, \mathtt{a}, \mathtt{aa}, \mathtt{aaa}, \ldots\}$.

$$(\mathtt{0} + \mathtt{1})^*$$

corresponds to all binary strings.

Example

An RE for the language of all binary strings of length at least 2 that begin and end in the same symbol.

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$$\mathtt{0}(\mathtt{0}+\mathtt{1})^*\mathtt{0} + \mathtt{1}(\mathtt{0}+\mathtt{1})^*\mathtt{1}$$

Note ***precedence*** of regular operators: *star* always refers to smallest piece it can, *or* to largest piece it can.

Example

Consider the regular expression

$$((\texttt{0}+\texttt{1})^*\texttt{1}+\varepsilon)\ (\texttt{0}\texttt{0})^*\ \texttt{0}\texttt{0}$$

Example

Consider the regular expression

$$((0+1)^*1+\varepsilon)\,(00)^*\,00$$

This RE is for the set of all binary strings that end with an even nonzero number of 0's.

Note that different language to:

$$(0+1)^*\,(00)^*\,00$$

Regular Operators for Languages

If one forms RE by the ***or*** of REs R and S, then result is union of R and S.

If one forms RE by the ***concatenation*** of REs R and S, then the result is all strings that can be formed by taking one string from R and one string from S and concatenating.

If one forms RE by taking the ***star*** of RE R, then the result is all strings that can be formed by taking any number of strings from the language of R (possibly the same, possibly different), and concatenating.

Regular Operators Example

If language L is $\{\mathtt{ma}, \mathtt{pa}\}$ and language M is $\{\mathtt{be}, \mathtt{bop}\}$, then

$L + M$ is $\{\mathtt{ma}, \mathtt{pa}, \mathtt{be}, \mathtt{bop}\}$;

LM is $\{\mathtt{mabe}, \mathtt{mabop}, \mathtt{pabe}, \mathtt{pabop}\}$; and

L^* is $\{\varepsilon, \mathtt{ma}, \mathtt{pa}, \mathtt{mama}, \ldots, \mathtt{pamamapa}, \ldots\}$.

Notation: If Σ is some alphabet, then Σ^* is the set of all strings using that alphabet.

An RE for Decimal Numbers

English: “Some digits followed maybe by a point and some more digits.”

RE:

$$(- + \varepsilon)\ \mathtt{D}\ \mathtt{D}^*\ (\varepsilon + \mathtt{.}\ \mathtt{D}^*)$$

where D stands for a digit.

Kleene's Theorem

Kleene's Theorem. *There is an FA for a language if and only there is an RE for the language.*

Proof (to come) is algorithmic.

Regular language is one accepted by some FA or described by an RE.

Applications of REs

- Specify piece of programming language, e.g. real number. This allows automated production of ***tokenizer*** for identifying the pieces.
- Complex search and replace.
- Many UNIX commands take regular expressions.

Practice

Give an RE for each of the following three languages:

1. All binary strings with at least one 0
2. All binary strings with at most one 0
3. All binary strings starting and ending with 0

Solutions to Practice

1. $(0+1)^*0(0+1)^*$
2. $1^*+1^*01^*$
3. $0(0+1)^*0+0$

In each case several answers are possible.

Summary

A regular expression (RE) is built up from individual symbols using the three Kleene operators: union ($+$), concatenation, and star (*). The star of a language is obtained by all possible ways of concatenating strings of the language, repeats allowed; the empty string is always in the star of a language.