C1. Compute, where legal, $A + B$, $AB$, and $BA$ for the following matrices:

\[
A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}
\]

C2. Compute, where legal, $3R^T$, $R^2$, and $RS$ for the following matrices

\[
R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 1 \\ 2 & -3 \\ 1 & 1 \end{bmatrix}
\]

C3. For two general square matrices $A$ and $B$ of the same size, simplify

\[
\frac{1}{2} ((2A^T + B)^T) - (-1)(-A^T - B^T)^T
\]

C4. A $2 \times 2$ matrix $A$ is called orange if $A^2 = I$. For example, the identity $I$ itself is orange.

(a) Give a matrix, other than the identity, that is orange and has two zero entries.

(b) Give a matrix that is orange and none of its entries is zero.

C5. Matrix $A$ is symmetric if $A = A^T$ and skew-symmetric if $A = -A^T$. Prove that every square matrix can be written as a sum of symmetric and a skew-symmetric matrix.

C6. Consider the matrix transform with matrix

\[
A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 1 & 1 & -2 & 7 \\ 0 & 3 & -2 & 3 \end{bmatrix}
\]

In each of the following, find a vector $x$ whose image under the transform is $b$, and determine if $x$ is unique.

(i) $b = (0, 0, 0)$
(ii) $b = (3, 5, 2)$
(iii) $b = (1, 1, 1)$

C7. In each of the following, draw vectors $u = (1, 1)$ and $v = (2, -1)$ and their image under the indicated transformation. Describe geometrically what the transformation does.

\[
T_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix} \quad T_3 = \begin{bmatrix} 2 & 6 \\ 0 & 2 \end{bmatrix}
\]
Some Solutions

C1. $A + B$ is not possible.

\[
AB = \begin{bmatrix} 3 \end{bmatrix}, \quad BA = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}
\]

C2. $R^2$ not possible.

\[
3R^T = \begin{bmatrix} 3 & 3 \\ 3 & 0 \\ 3 & -6 \end{bmatrix}, \quad RS = \begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}
\]

C3. $\frac{1}{2}B^T - B$.

C4. (a) For example \[
\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}
\]

(b) For example \[
\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}
\]

C5. Use the fact that $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric and note that

\[
A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T).
\]

C6. (i) Many answers: e.g. $(0, 0, 0, 0)$

(ii) Many answer: e.g. $(13/3, 2/3, 0, 0)$

(iii) No solution

C7. $T_1$ maps $u$ to $(0, 1)$ and $v$ to $(0, -1)$. It projects onto $y$-axis.

$T_2$ maps $u$ to $(0, \sqrt{2})$ and $v$ to $(\frac{3}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. It rotates through 45 degrees.

$T_3$ maps $u$ to $(8, 2)$ and $v$ to $(-2, -2)$. It expands and shears.