Projections
**Fact.** If $W$ is a subspace of $V$, then every vector $y$ in $V$ can be written uniquely as the sum of a vector in $W$ and a vector in $W^\perp$.

We will call the vector in $W$ the projection of $y$ onto $W$. 
Consider in $\mathbb{R}^3$ the plane $P$ given by $3x+4y-z = 0$. Say we want $v = (9, 9, 11)$ as the sum of vector in $P$ and vector in $P^\perp$. We will see an algorithm below.

But for ad hoc approach: we know that any vector in $P^\perp$ is multiple of $w = (3, 4, -1)$. So if we assume the requisite vector in $P^\perp$ is $aw$, then we need $(v - aw) \cdot v = 0$. This solves to $a = 2$; so $v = 2w + (3, 1, 13)$.
**Defn.** The (orthogonal) projection of vector $y$ onto vector $u$ is its “shadow”. It is denoted by $\text{proj}_u(y)$. 
**Fact.** For vectors \( y \) and \( u \), the projection of \( y \) onto \( u \) is given by:

\[
\text{proj}_u(y) = \frac{y \cdot u}{u \cdot u} u
\]
Example Projection

If \( a = (3, 4) \) and \( b = (-5, 2) \) then
\[
\operatorname{proj}_b(a) = \left(\frac{35}{29}, -\frac{14}{29}\right) \quad \text{and}
\]
\[
\operatorname{proj}_a(b) = \left(-\frac{21}{25}, -\frac{28}{25}\right).
\]
**Projection onto a Subspace**

**Defn.** The (orthogonal) projection $\text{proj}_W(y)$ of the vector $y$ onto the vector space $W$ is vector in $W$ such that $y - \text{proj}_W(y)$ in $W^\perp$.

**Fact.** If we think of $y$ as a point, then the projection of it onto $W$ is the closest point of $W$ to it.
**Fact.** If $W$ is a subspace with orthonormal basis $\{w_i\}$, then

$$\text{proj}_W(y) = \sum_i (y \cdot w_i) w_i$$
If $W$ is a subspace of $V$, then every vector $y$ in $V$ can be written uniquely as the sum of a vector in $W$ and a vector in $W^\perp$. The (orthogonal) projection $\text{proj}_W(y)$ of $y$ onto $W$ is the vector in $W$ such that $y - \text{proj}_W(y)$ in $W^\perp$. Equivalently, the projection is the closest point of $W$ to $y$. 
For vectors $y$ and $u$, the projection of $y$ onto $u$ is given by: $\text{proj}_u(y) = \frac{y \cdot u}{u \cdot u} u$. If $W$ is a subspace with orthonormal basis $\{w_i\}$, then $\text{proj}_W(y) = \sum_i (y \cdot w_i) w_i$. 

Summary (cont)