The Dot Product and Orthogonal Vectors
The Dot Product

Defn. The **dot product** (or **inner product**) of two vectors $u$ and $v$ denoted $u \cdot v$ is the sum of the product of corresponding entries.

That is

$$u \cdot v = \sum_i u_i v_i.$$
For example,
\[
\begin{bmatrix}
3 \\
-1 \\
-2
\end{bmatrix} \cdot \begin{bmatrix}
4 \\
4 \\
7
\end{bmatrix} = 12 - 4 - 14 = -6.
\]

If we view the two vectors as matrices, then the dot product $u \cdot v$ is the entry in the $1 \times 1$ matrix given by $u^T v$.
Several facts one could write. These include the **distributive law**:

\[ u \cdot (v + w) = u \cdot v + u \cdot w \]
Dot Product and Orthogonal Vectors

Defn. Two vectors are **orthogonal** if their dot product is zero.

Orthogonal vectors are sometimes called **perpendicular** vectors.
**Norm of a Vector**

**Defn.** The length (or norm) of vector $v$ is $||v|| = \sqrt{v \cdot v}$. A unit vector has length 1.
Normalization

**ALGOR** To obtain a unit vector in the same direction, divide by the length.

For example, the vector \((3, -1, 2)\) has norm \(\sqrt{14}\); a unit vector in the same direction is \(\frac{1}{\sqrt{14}}(3, -1, 2)\).
Fact. Pythagoras’ Theorem
Vectors $u$ and $v$ are orthogonal if and only if $||u + v||^2 = ||u||^2 + ||v||^2$.

Proof is by computation.
Summary

The dot product $u$ and $v$ of two vectors $u \cdot v$ is the sum of the product of corresponding entries. As matrices, the dot product is (the entry in) the matrix $u^T v$.

Two vectors are orthogonal if their dot product is zero. The length/norm of vector $v$ is $\sqrt{v \cdot v}$. To obtain a unit vector in the same direction, divide by the length.

Pythagoras’ Theorem is that $u$ and $v$ are orthogonal if and only if $||u + v||^2 = ||u||^2 + ||v||^2$. 

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