Recursion

Often in solving a problem one breaks up the problem into subtasks. Recursion can be used if one of the subtasks is a simpler version of the original problem.

6.1 An Example

Suppose we are trying to sort a list of numbers. We could first determine the minimum element; and what remains to be done is to sort the remaining numbers. So the code might look something like this:

```cpp
void sort(Collection &C) {
    min = C.getMinimum();
    cout << min;
    C.remove(min);
    sort(C);
}
```

Every recursive method needs a stopping case: otherwise we have an infinite loop or an error. In this case, we have a problem when C is empty. So one always checks to see if the problem is simple enough to solve directly.

```cpp
void sort(Collection &C) {
    if( C.isEmpty() )
        return;
    ... // as before
}
```

Example. Printing out a decimal number. The idea is to extract one digit and then recursively print out the rest. It’s hard to get the most significant digit, but one can obtain the least significant digit (the “ones” column): use num % 10. And then num/10 is the “rest” of the number.

```cpp
void print( int n ) {
    if( n>0 ) {
        print ( n/10 );
        cout << n%10 ;
    }
}
```
6.2 Tracing Code

It is important to be able to trace recursive calls: step through what is happening in the execution. Consider the following code:

```c++
void g( int n ) {
    if( n==0 ) return;
    g(n-1);
    cout << n;
}
```

It is not hard to see that, for example, \( g(3) \) prints out the numbers from 3 down to 1. But, you have to be a bit more careful. The recursive call occurs before the value 3 is printed out. This means that the output is from smallest to biggest.

```
1
2
3
```

Here is some more code to trace:

```c++
void f( int n ) {
    cout << n;
    if(n>1)
        f(n-1);
    if(n>2)
        f(n-2);
}
```

If you call the method \( f(4) \), it prints out 4 and then calls \( f(3) \) and \( f(2) \) in succession. The call to \( f(3) \) calls both \( f(2) \) and \( f(1) \), and so on. One can draw a recursion tree: this looks like a family tree except that the children are the recursive calls.

```
\begin{center}
f(4)
\hspace{1cm}
f(3)\hspace{1cm}f(2)
\hspace{1cm}
f(2)\hspace{1cm}f(3)\hspace{1cm}f(1)
\hspace{1cm}
f(1)\hspace{1cm}f(1)\hspace{1cm}f(1)
\hspace{1cm}
f(1)
\end{center}
```

Then one can work out that \( f(1) \) prints 1, that \( f(2) \) prints 21 and \( f(3) \) prints 3211. What does \( f(4) \) print out?
Exercise
Give recursive code so that \texttt{brackets(5)} prints out ((((()))).

6.3 Methods that Return Values
Some recursive methods return values. For example, the sequence of Fibonacci numbers
1, 1, 2, 3, 5, 8, 13, 21, \ldots is defined as follows: the first two numbers are 1 and 1,
and then each next number is the sum of the two previous numbers. There is obvious
recursive code for the Fibonacci numbers:

\begin{verbatim}
int fib(int n) {
  if( n<2 )
    return 1;
  else
    return fib(n-1) + fib(n-2);
}
\end{verbatim}

Warning: Recursion is often easy to write (once you get used to it!). But occa-
sionally it is very inefficient. For example, the code for \texttt{fib} above is terrible. (Try to
calculate \texttt{fib(30)}.)

6.4 Application: The Queens problem
One can use recursion to solve the \textit{Queens} problem. The old puzzle is to place 8 queens
on a $8 \times 8$ chess/checkers board such that no pair of queens attack each other. That
is, no pair of queens are in the same row, the same column, or the same diagonal.

The solution uses search and \textit{backtracking}. We know we will have exactly one
queen per row. The recursive method tries to place the queens one row at a time. The
main method calls \texttt{place(0)}. Here is the code/pseudocode:

\begin{verbatim}
void placeQueen(int row) {
  if(row==8)
    celebrateAndStop();
  else {
    for( queen[row] = all possible vals ) {
      check if new queen legal;
      record columns and diagonals it attacks;
      // recurse
      placeQueen(row+1);
      // if reach here, have failed and need to backtrack
    }
  }
}\end{verbatim}
6.5 Application: The Steps problem

One can use recursion to solve the Steps problem. In this problem, one can take steps of specified lengths and has to travel a certain distance exactly (for example, a cashier making change for a specific amount using coins of various denominations).

The code/pseudocode is as follows

```cpp
bool canStep(int required)
{
    if( required==0 )
        return true;
    for( each allowed length )
        if( length<=required && canStep(required-length) )
            return true;
    //failing which
    return false;
}
```

The recursive boolean method takes as parameter the remaining distance required, and returns whether this is possible or not. If the remaining distance is 0, it returns true. Else it considers each possible first step in turn. If it is possible to get home after making that first step, it returns true; failing which it returns false. One can adapt this to actually count the minimum number of steps needed. See code below.

One can also use recursion to explore a maze or to draw a snowflake fractal.

Sample Code

`StepsByRecursion.cpp`