1. Give three disjoint bases for \( \mathbb{R}^2 \).

2. In a paragraph or two, outline the proof that for a finite-dimensional vector space \( V \) and operator \( T: V \to W \) it holds that \( \dim V = \dim \text{null } T + \dim \text{range } T \).

3. True or false:
   (a) For any subspace \( U \) it holds that \( U + U = U \).
   (b) \( \mathbb{C}^4 \) is isomorphic to \( \mathbb{R}^4 \).
   (c) For any vector space \( V \): If \( S, T \in \mathcal{L}(V) \), and \( S, T \) both invertible, then \( ST \) invertible.
   (d) For any vector space \( V \): If \( S, T \in \mathcal{L}(V) \), and \( ST \) invertible, then \( S \) and \( T \) both invertible.

4. Give an example of spaces \( V \) and \( W \) such that \( \dim \mathcal{L}(V, W) = 5 \).

5. Consider the vector space \( \mathcal{P}_2[\mathbb{R}] \). Let \( D \) be the operator differentiation (for example \( D(3x^2 + 5) = 6x \)).
   (a) Give a basis for \( \text{null } D \) and \( \text{range } D \).
   (b) Determine the matrix of \( D \) with respect to the standard basis \( (1, x, x^2) \).
   (c) Explain what that tells us about the eigenvalues of \( D \).
   (d) Determine the matrix of \( D \) with respect to the basis \( (1 + x^2, 1 + x, x + x^2) \).

6. (a) Give an example of a space and operator where every nonzero vector is an eigenvector.
    (b) Give an example of a space and operator where no nonzero vector is an eigenvector.