• Axler 10.20, 10.22

• Let $T_n$ be the transitive complete digraph on $n$ vertices. This is the digraph formed by taking $n$ vertices and numbering them from 1 up to $n$, and adding an arc between every pair of vertices oriented from the smaller vertex to the larger vertex. For example, here is $T_3$.

Calculate the eigenvalues and eigenvectors of $T_n$.

• Let $G$ be an undirected graph with the following properties: $r$-regular; adjacent vertices do not have a common neighbor (that is, no 3-cycles); every pair of nonadjacent vertices have exactly 3 common neighbors.

  (a) Explain why we get the matrix equation $3A + A^2 - (r - 3)I = 3J$.
  (b) Explain why $G$ must have at least $1 + r + r(r - 1)/3$ vertices.
  (c) Show that if $G$ has exactly the minimum number of vertices given in (b), then $r = 3$.
  (d) Draw $G$ as found in (c).

Due: Friday October 16