Answer any three of the following:

1. Attend the talk “Optimal Control Strategies for Epidemics on Heterogeneous Networks” by Krishna Pacifici (Thursday, Sept 25, 4:30 - 5:30 pm in Martin M-102). Write a 1–2 page summary of the talk, focusing on the use of models.

2. Consider the dynamical system generated by the logistics equation \( g(x) = rx(1 - x) \). In class we saw that for \( r \approx 3.84 \) there is an attractive oscillation of period 3.
   - Using a computer and/or calculations, determine the \( r \) where this period 3 oscillation emerges. (Say to 4 decimal places.)
   - It is known that at this value of \( r \) that \( g^{(3)}(x) \) is tangent to the line \( y = x \) at three points. Why does this happen?

3. Consider the dynamical system given by the recurrence:
   \[
   x_{n+1} = \begin{cases} 
   rx_n^{1-b} & x_n > K \\
   rx_n & x_n \leq K 
   \end{cases}
   \]
   (a) Draw a typical cobweb plot.
   (b) Determine the fixed points and their stability.
   (c) Determine the first bifurcation value.

4. A dominating set in a graph is a set \( S \) of vertices such that every vertex not in \( S \) has some neighbor in \( S \). The domination number is the minimum size of a dominating set.
   (a) The random graph \( G(n, \frac{1}{2}) \) is known to have domination number about \( \log_2 n \). Sketch a convincing argument for this.
   (b) Using Sage (or other software), get some data on the domination number of \( H(n, 2) \). Formulate a conjecture.

5. (From Bonato) Determine which of the following graphs are power law graphs. Justify your answer.
   (a) The highway graph: Vertices are cities, while edges correspond to roads between them.
   (b) The airport graph. Vertices are airports, and edges are airline routes.
   (c) The musician graph. Vertices are musicians, and edges correspond to having played together in a concert or recording.
   (d) The word graph. Vertices are words in the English language, and two words are joined if they appear in the same sentence.
   (e) The art gallery graph. Vertices are art galleries, and two galleries are joined if they display art by the same artist.

Due: START OF CLASS Thursday October 2.
Further, you will need to present your answers to one of two questions.