1. Here is solution: $t(n) = 3^n + 5 \cdot (-2)^n$. Give the recurrence.

\[
t(n) = t(n-1) + 6 \cdot t(n-2)
\]
\[
t(0) = 6
\]
\[
t(1) = -7
\]

2. Let $b(n)$ be the number of binary strings of length $n$ that do not contain three 1's in a row. Give a recurrence for $b(n)$.

\[
b(n) = b(n-1) + b(n-2) + b(n-3)
\]

Start: 0, 0, 1, 2, \ldots
3. Call a rooted tree *gorgeous* if every vertex has an even number of children. Draw all gorgeous rooted trees with 7 vertices, assuming vertices are indistinguishable and the order of children doesn’t matter.

4. Consider the graph obtained from a cycle with $2m$ vertices ($m \geq 3$) by joining all pairs of diametrically opposite vertices, so that every vertex has degree 3.

(a) What is this graph called for $m = 3$?

(b) For what $m$ is this graph planar?

(c) What is the chromatic number for $m = 100$?

Brooks $\Rightarrow \chi \leq \max \text{ degree}$

And graph has odd cycle.