Simultaneous Games
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Consider game where two players make choice simultaneously. There is some rule for a winner. E.g. Rock, Paper, Scissors

Or more generally, there is a rule for a payout from one player to the other. Can be represented as a matrix.
Example: Thumbs

Say players Matcher and Opposer. Both players simultaneously produce either one thumb or two.

If number of thumbs equal, Matcher wins total number of thumbs in dollars from other player. Otherwise, Opposer wins total number of thumbs in dollars.
Thumbs as a Matrix

The game from MATCHER’s perspective as row-chooser:

<table>
<thead>
<tr>
<th></th>
<th>Two</th>
<th>One</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two</td>
<td>+4</td>
<td>−3</td>
</tr>
<tr>
<td>One</td>
<td>−3</td>
<td>+2</td>
</tr>
</tbody>
</table>
Minimax Theorem (John von Neumann)

The optimal strategies are always to play each option with a certain fixed probability.

Moreover, solution is stable in that if one player *unilaterally* deviates, then they can only do worse.

(The strategies and the value of the game can be calculated by linear programming.)
Example: Thumbs

Each player should do two thumbs with probability $\frac{5}{12}$.

OPPOSER’s expected return $\frac{1}{12}$ of a dollar.
Final Jeopardy

Players simultaneously bet some of their money on a double-or-nothing question.
Example Final Jeopardy

Say: Alejandro has $10,000 and Brianna has $7,000.

Alejandro need never bet more than $4,001. But if so, Brianna should bet small! But if so, Alejandro should bet small! But if so, Brianna should bet everything!
Actual answer: both players have option of ’Small’ and ’Large’ bet.

Should choose randomly with some proportions determined by odds of getting question right. . .
Non-Zero Sum Games

Things change if games not zero-sum (meaning losses of one equals winnings of other). They sometimes have stable “solutions” called Nash equilibria.

But even then, things are strange. Famous example is *Prisoner’s Dilemma*. 
More than Two Players

There is no optimal solution. Can be at the whim of the weaker players. Psychology matters.