Math 4190 — Goddard — Fall18
Assignment 9

You may work in pairs, and submit one answer sheet for the pair.

1. Define the graph $G_m$ as the graph that is a grid of $3m$ vertices arranged in 3 rows and $m$ columns such that each vertex has an edge to the vertices to the left, above, to the right, and below it, if they exist. For example, $G_4$ is illustrated here.

![Image of graph $G_4$]

Complete the following, with justifications:
(a) $G_m$ has an Euler tour if and only if
(b) $G_m$ has a Hamilton cycle if and only if

2. A tournament is obtained by taking the complete graph $K_n$ and orienting every edge to form a directed graph (where every road is a one-way street).
(a) Show that a tournament always has a directed Hamilton path.
(b) Show that a tournament might not have a directed Hamilton cycle.

3. The prism of a graph $G$ on $n$ vertices is obtained by taking two separate copies of $G$ and adding $n$ “parallel” edges joining the corresponding vertices. For example, the prism of the hypercube $Q_n$ is $Q_{n+1}$. State and prove the relationship between the chromatic number of $G$ and the chromatic number of its prism.

4. Calculate the chromatic number of the following graph:

![Image of graph]

5. Show that $K_{2,m}$ is planar for all $m$.

6. For the general Hamming code with $k$ check bits, show that the distance is exactly 3.

7. (a) Consider a code with distance $d$ with $d$ odd. Show that if one appends a parity bit to every string, then the new code has distance $d + 1$.
(b) Give an example that shows that part (a) is not necessarily true if $d$ is even.

Due: Start of class Wednesday 28 November

Game of the Week. Kudu biltong.