Assignment 8

You may work in pairs, and submit one answer sheet for the pair.

1. Solve the recurrence \(h(n) = 6h(n - 1) - 8h(n - 2)\), with \(h(1) = 4\) and \(h(2) = 16\), by using the characteristic equation.

2. Prove that a cycle has a bipartite coloring if and only if the number of vertices is even.

3. Draw a rooted tree with 5 vertices labeled \(A\), \(B\), \(C\), \(D\), and \(E\) such that all of the following conditions hold:
   (i) \(E\) has exactly two ancestors
   (ii) \(A\) and \(C\) are siblings
   (iii) No vertex has exactly one child
   (iv) \(B\) is neither an ancestor nor a descendant of \(C\)
   (v) \(B\) is not the grandparent of \(D\)

4. (a) Show that there are exactly two trees on 4 vertices if the vertices are indistinguishable.
   (b) How many different trees are there with 4 vertices if the 4 vertices are all distinguishable (say the vertices are labeled \(A, B, C, D\))?
   (c) How many different rooted trees are there with 4 vertices if the 4 vertices are indistinguishable (and the ordering of children does not matter)?
   (d) How many different rooted trees are there with 4 vertices if the 4 vertices are all distinguishable (but the ordering of children does not matter)?

5. Draw all (simple) graphs with 5 vertices and 7 edges.

6. Just as a domino is two cells stuck together, a tetromino is four cells stuck together. For example, the game Tetris is played with falling tetrominoes. To tile a square means covering the whole square with no overlap.
   (a) Draw the five different tetrominoes (we consider here a tetromino the same as its mirror image).
   (b) Show that four of these tetrominoes each have the property that they can tile a \(4 \times 4\) square.
   (c) Shown that none of these can tile a \(5 \times 5\) square.
   (d) Determine which of these can tile a \(6 \times 6\) square.

Due: Monday 12 November

Game of the Week. Consider the following game. There are three piles of matches, with sizes 3, 5, and 7. Each player takes turns to choose a pile and remove some or all of the matches in that pile. The winner is the one who takes the last match.