Math 4190 — Goddard

In-class Practice 30: Graphs

Consider the following graph \( R_m \). This graph has \( 2m^2 \) vertices. Each vertex is given by an ordered pair \((a, b)\) with \( a \in \{1, 2, \ldots, m\} \) and \( b \in \{1, 2, \ldots, 2m\} \). Two vertices \((a, b)\) and \((c, d)\) are connected by an edge if and only if they agree in one coordinate; that is, if \( a = c \) or \( b = d \). For example, \( R_2 \) is drawn.

(a) For what \( m \) does \( R_m \) have a Euler tour?

The degree of every vertex is \((m - 1) + (2m - 1) = 3m - 2\).
So Euler tour iff \( m \) is even.

(b) For what \( m \) does \( R_m \) have a Hamilton cycle?

All \( m \geq 2 \).