In-class Practice 24: The Fibonacci Numbers

(a) Write down the first 10 Fibonacci numbers (starting with $f(0) = f(1) = 1$).

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

(b) Prove that the Fibonacci numbers obey the equation:

$$2f(m) + 3f(m+1) = f(m+4)$$

Base case $m = 0$: LHS = $2 + 3 = 5$ = RHS

General case: LHS = $2f(m) + 3f(m + 1) = 2f(m) + 3(f(m) + f(m - 1)) = (3f(m) + 2f(m - 1)) + (2f(m) + f(m - 1)) \overset{IH}{=} f(m + 3) + (f(m) + f(m + 1)) = f(m + 3) + f(m + 2) = f(m + 4) = RHS$