Prove by contradiction that if there are 999 people in a room, at least three must share a common birthday.

Suppose it is not true that at least three share a common birthday. Then for each day of the year there are at most two people with that birthday. That is, there are at most $2 \times 366$ people. But that number is less than 999, a contradiction.