1. Define two Hamilton cycles to be the same if they use the same edges. How many Hamilton cycles are there in $K_{2021}$?

\[ 2020! / 2 \]

2. The grid graph $G_m$ has $m^2$ vertices arranged into a square grid with each vertex having an edge to each of the vertex to the left, to the right, above and below (if they exist).

(a) For which $m$ is $G_m$ bipartite?
(b) For which $m$ does $G_m$ have a Hamilton cycle?
(c) For which $m$ is $G_m$ planar?

(a) All $m$ (has checkerboard coloring)
(b) $m$ even (need equal numbers of each color)
(c) All $m$ (just draw it)

3. Consider the graph obtained from a cycle with $2m$ vertices ($m \geq 3$) by joining all pairs of diametrically opposite vertices (so that every vertex has degree 3).

(a) What is this graph called for $m = 3$?
(b) For what $m$ is this graph planar?
(c) What is the chromatic number for $m = 2021$?

(a) $K_{3,3}$; (b) No $m$ (always “contains” $K_{3,3}$);
(c) 2 (if we alternate colors on the cycle, then added edges join vertices of opposite colors)

4. For the code with the following generator matrix:

\[
\begin{pmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1
\end{pmatrix}
\]

(a) How many strings in the code?
(b) What is the distance?
(c) Is the code 1-error-detecting?
(d) Is the code 1-error-correcting?

(a) $2^3$ (b) 2 (helps to list them) (c) Yes (d) No