Warmup 5: Trees, Graphs, and Degrees

1. Complete the following.
   (a) A recursive definition of a rooted tree is that (i) a single root is a rooted tree, and
       (ii) adding ________________________ yields a rooted tree.
   (b) If a graph has the property that between every two vertices there is a _______ path, then it is a tree.
   (c) One definition of a tree is that it is a graph that is _______ and contains ________
   (d) A tree with 2021 vertices has ________ edges.

2. Consider graphs with 8 vertices that have degree sequence 4, 2, 2, 2, 1, 1, 1, 1.
   (a) Draw a tree with such a degree sequence.
   (b) Draw a graph with such a degree sequence that is not a tree.

3. Call a rooted tree gorgeous if every vertex has an even number of children. Draw all gorgeous rooted trees with 7 vertices, assuming vertices are indistinguishable and the order of children doesn’t matter.

4. For $m \geq 1$, the doubleFan graph $D_m$ is defined by taking a path with $m$ vertices and adding two new vertices and joining each new vertex to each vertex on the path. For example, the graph $D_3$ is shown here.

   (a) For what values of $m$ does the doubleFan $D_m$ have an Euler tour?
   (b) For what values of $m$ does the doubleFan $D_m$ have an Euler trail?