A **rooted tree** has **vertices** and **edges** with one vertex designated the **root** and all other vertices having a unique parent.

Rooted trees are normally drawn with the root at the top.

A **tree** is just like a rooted tree, except it does not have a special vertex.
Properties of Trees

(1) A tree is connected

(2) A tree contains no cycle

(3) Between any two vertices in a tree there is a unique path.

(4) If a tree has $n$ vertices, then it has $n - 1$ edges.
A (simple) graph is a collection of vertices and edges such that each edge joins two vertices.

We do NOT allow multiple edges nor self-loops.
A **walk** is a sequence of vertices such that consecutive vertices are joined by an edge.

The **length** of a walk is the number of edges.

A **path** is a walk without repeated vertices.

A **cycle** is a walk without repeated vertices except that the first and last vertex are the same.

The terms **path** and **cycle** also refer to specific graphs.
A graph is *connected* if between every two vertices there is a walk.
The **degree** of a vertex is the number of edges coming out of it.

In a graph, the sum of the degrees is twice the number of edges.
The **complete graph** $K_n$ on $n$ vertices has every pair of vertices joined by an edge.

The **complete bipartite graph** $K_{r,s}$ has $r$ vertices on one side, and $s$ vertices on the other. All edges go between the two sides.
An **Euler tour** is a walk that goes along every edge exactly once, and ends up where one started.

A connected graph has an Euler tour if and only if every vertex has even degree.