1. Prove that every positive integer can be written as a sum of some distinct Fibonacci numbers with the added restriction that no two of the Fibonacci numbers used are consecutive. For example, $28 = f(7) + f(4) + f(2)$.

2. (a) Show that there are exactly two trees on 4 vertices if the vertices are indistinguishable.
   
   (b) How many different trees are there with 4 vertices if the 4 vertices are all distinguishable (say the vertices are labeled A, B, C, D)?

   (c) How many different rooted trees are there with 4 vertices if the 4 vertices are indistinguishable (and the ordering of children does not matter)?

   (d) How many different rooted trees are there with 4 vertices if the 4 vertices are all distinguishable (but the ordering of children does not matter)?

3. Consider a simple graph with 100 vertices.
   
   (a) Explain why such a graph has at most $\binom{100}{2}$ edges.

   (b) Describe such a graph that has $\binom{99}{2}$ edges but is not connected.

   (c) Prove that if such a graph has more than $\binom{99}{2}$ edges then it is connected.

4. In both the following cases, draw a tree with that degree sequence or prove that it is impossible:

   (a) 4, 3, 3, 2, 1, 1, 1, 1

   (b) 4, 3, 3, 2, 1, 1, 1, 1

Due: 10:10am Wednesday 3 November