You may work in pairs, and submit one answer sheet for the pair.

1. Show that the product of four consecutive integers is a multiple of 12.

2. Give a proof by contradiction that if $x, y, z$ are integers such that $xyz \leq 1000$, then at least one of $x, y, z$ is at most 10.

3. Let $p$ be a prime number other than 2. Prove that $2p$ cannot be written as the difference of squares (of integers).

4. Calculate the gcd of:
   
   (a) 91 and 287.
   
   (b) $12^{100}$ and $100^{12}$.

5. Give the multiplication tables for $\mathbb{Z}_5$ and $\mathbb{Z}_8$.

6. Consider $\mathbb{Z}_{10}$.
   
   (a) List all elements of $\mathbb{Z}_{10}$.
   
   (b) What is the inverse of 3?
   
   (c) Give all square-roots of 6. (That is, all elements whose square is 6.)
   
   (d) How many rows of the multiplication table contain every element?

Due: 10:10am Wednesday 29 September