Math 4190 — Goddard — Fall21

Assignment 2

You may work in pairs, and submit one answer sheet for the pair.

1. An SSN is a 9-digit number with zeroes allowed in every position.
   (a) How many SSNs have exactly two distinct digits?
   (b) How many SSNs have digits that sum to 2?
   (c) How many SSNs have digits that sum to 3?

2. Consider three balls and three buckets. In how many different ways can the balls be arranged in the buckets if:
   (a) the balls and the buckets are all numbered?
   (b) the balls are numbered but the buckets are indistinguishable?
   (c) the buckets are numbered but the balls are indistinguishable?
   (d) the balls are indistinguishable and the buckets are indistinguishable?

3. In the local lottery, you buy a ticket with 6 (unordered) numbers in the range 1 to 49, and you have to match the 6 numbers drawn to win the jackpot.
   (a) Calculate \( \binom{49}{6} \).
   (b) A runner-up prize is obtained if you match exactly 5 of the drawn numbers. Calculate the odds of a runner-up prize.
   (c) All tickets that match no numbers are placed in a barrel for a chance at a “lucky loser” prize. Calculate the odds of a particular ticket matching no numbers.

4. Show that if \( p \) is a prime number, then \( \binom{p}{i} \) is a multiple of \( p \) for all \( i \) from 1 up to \( p - 1 \).

Due: 10:10 Wednesday 1 September