You may work in pairs, and submit one answer sheet for the pair.

1. (a) Suppose that there are 6 people in a room. Show that one can always find a group of 3 people such that either nobody in the group knows anybody in the group or everybody in the group knows everyone in the group.
   (b) Show that this conclusion does not hold if there are only 5 people in the room.

2. For the general Hamming code with \( k \) check bits, show that the distance is exactly 3.

3. Show that the distance of the Reed–Muller code \( R_m \) is exactly \( 2^{m-1} \).

4. Consider the Reed–Muller code \( R_3 \).
   (a) List all strings in \( R_3 \).
   (b) Provide a suitable generator matrix.

5. In the gym there is a row of 100 lockers. The students come in one at a time. The first student goes to each locker and opens it. The second student goes to every second locker starting with the second, and closes it. The third student goes to every third locker starting with the third, and closes it if open and opens it if closed. The fourth student goes to every fourth locker starting with the fourth, and toggles the state of it. And so on. After all 100 students have come in, how many lockers are open?

Due: 6pm Tuesday 30 November SHARP