1. Give, with justification, a formula for the minimum number of edges that must be added to a general connected graph to make it have an Euler tour, provided we allow multiple edges (that is, two vertices can be joined by more than one edge).

2. Does the following graph have a Hamilton path? A Hamilton cycle?

![Graph Image]

3. The wheel $W_k$ is obtained from the cycle on $k$ vertices by adding one new vertex connected to all other vertices. Calculate the chromatic number of a wheel.

4. Show that the hypercube $Q_4$ is not planar.

5. Assume $n$ and $a_1, \ldots, a_k$ are positive integers. Then the graph $Z_n[a_1, \ldots, a_k]$ is defined as follows. The vertex set is $Z_n$. For each vertex $x$ and each $a_i$, there is an edge from $x$ to $x + a_i$ (with arithmetic in $Z_n$). For example, here is $Z_9[1,3]$.

![Graph Image]

(a) Draw $Z_6[2,3]$.
(b) When is $Z_{2021}[a_1, \ldots, a_k]$ bipartite?
(c) When is $Z_{2021}[a_1]$ connected?
(d) When is $Z_{2021}[a_1, a_2]$ connected?

Due: 10:10am Wednesday 17 November