You may work in pairs, and submit one answer sheet for the pair.

1. Consider a board like a checkerboard that is partitioned into squares. Define a tromino tiling of a board to mean covering the board completely with nonoverlapping trominoes, where each tromino covers three squares in a row (horizontally or vertically). Let \( t(n) \) be the number of tromino tilings of the \( 3 \times n \) board. Give a recurrence formula for \( t(n) \).

2. Prove that the Fibonacci sequence obeys the following identity:
\[
f(0) + f(1) + \ldots + f(n) = f(n + 2) - 1.
\]

3. Prove that the Fibonacci numbers obey the following identity:
\[
\sum_{i=0}^{n} [f(i)]^2 = f(n)f(n + 1) \quad \text{for} \quad n \geq 0.
\]

4. Prove that every positive integer can be written as a sum of some distinct Fibonacci numbers with the added restriction that no two of the Fibonacci numbers used are consecutive. For example, \( 28 = f(7) + f(4) + f(2) \).

5. (a) Show that any two consecutive Fibonacci numbers \( f(n-1) \) and \( f(n) \) are relatively prime.

(b) Code up or apply the Extended Euclid algorithm to find \( s \) and \( t \) such that \( sf(n) + tf(n-1) = 1 \). Discuss your results, conjecture a pattern, and try to prove your conjecture.

Due: Monday 2 April

Game of the Week. Consider the following game. There are three piles of matches, with sizes 3, 5, and 7. Each player takes turns to choose a pile and remove some or all of the matches in that pile. The winner is the one who takes the last match.