(This is an individual assignment)

1. Give the general solution of the recurrence \( g(n) = 2g(n - 1) + 3g(n - 2) \) by using the characteristic equation.

2. Consider the recurrence \( h(n) = 4h(n - 1) - 4h(n - 2) \) \((n \geq 2)\).
   
   (a) Show that the characteristic equation has only one root.
   
   (b) Show that both \( h(n) = 2^n \) and \( h(n) = n2^n \) are solutions to the recurrence.
   
   (c) Suppose \( h(0) = 1 \) and \( h(1) = 2 \). Solve the recurrence.

3. Consider a simple graph with 100 vertices.
   
   (a) Explain why such a graph has at most \( \binom{100}{2} \) edges.
   
   (b) Describe such a graph that has \( \binom{99}{2} \) edges but is not connected.
   
   (c) Prove that if such a graph has more than \( \binom{99}{2} \) edges then it is connected.

4. In both the following cases, draw a tree with that degree sequence or prove that it is impossible:
   
   (a) 4, 3, 3, 2, 1, 1, 1, 1, 1
   
   (b) 4, 3, 3, 2, 1, 1, 1, 1, 1

5. The Wheel graph \( W_n \) is obtained by taking a cycle with \( n \) vertices and adding one new vertex that is joined to every vertex in the cycle. Complete the following, with justifications:
   
   (a) \( W_n \) has an Euler tour if and only if
   
   (b) \( W_n \) has a Hamilton cycle if and only if

6. A tournament is obtained by taking the complete graph \( K_n \) and orienting every edge to form a directed graph (where every road is a one-way street).
   
   (a) Show that a tournament always has a directed Hamilton path.
   
   (b) Show that a tournament might not have a directed Hamilton cycle.

7. An outerplanar graph is a graph that can be drawn in the plane with all its vertices on the outside.
   
   (a) Determine which complete graphs are outerplanar.
   
   (b) Determine which complete bipartite graphs are outerplanar.
   
   (c) Explain why it follows from the Four-Color Theorem that every outerplanar graph has chromatic number at most 3.

Due: Start of class Tuesday 14 November