You may work in pairs, and submit one answer sheet for the pair.

1. Take $p = 11$, $q = 13$ and $a = 37$ in RSA. Verify that $b = 13$. If $M = 62$, what does Alice send?

2. Prove by induction that for all $n \geq 1$

\[
\sum_{i=1}^{n} i 2^i = (n - 1)2^{n+1} + 2.
\]

3. Prove by induction a formula for $\sum_{i=1}^{n} i^3$. (Hint: it is a polynomial of degree 4.)

4. Prove by induction that for every positive integer $n$, $4^{2n+1} + 11^n$ is divisible by 5.

5. Consider a prison which is the shape of an $n$-gon (not necessarily convex). The warden has three teams of guards, a red team, a blue team, and a green team. The warder wants to assign each vertex of the polygon to one (guard of a) specific team, such that every point in the interior of the polygon is visible to (at least one member of) each team.

   (a) Prove by induction that this is possible.
   
   (b) Construct a 100-gon prison where, if we have to leave one of the vertices unassigned, then it is not possible to satisfy the warden’s requirements.

Due: Start of class Monday 12 March

**Game of the Week.** Consider the following game. A number of X’s are drawn on a straight line. Two players take turns in joining two consecutive X’s, except that an X may be used only once. The last person to be able to draw is the winner. How to play well?