1. Just as a domino is two squares stuck together, a tetromino is four cells stuck together. For example, the game Tetris is played with falling tetrominoes. To tile a square means covering the whole square with no overlap.

   (a) Draw the five different tetrominoes (we consider here a tetromino the same as its mirror image).

   (b) Show that four of these tetrominoes each have the property that they can tile a $4 \times 4$ square.

   (c) Shown that none of these can tile a $5 \times 5$ square.

   (d) Determine which of these can tile a $6 \times 6$ square.

2. Let $a$ be a positive integer. Prove that $2^a + 1$ and $4^a + 1$ are relatively prime.

3. Determine gcd($10^{10}$, $20^{20}$).

4. Consider $\mathbb{Z}_{10}$.

   (a) List all elements of $\mathbb{Z}_{10}$.

   (b) What is the inverse of 3?

   (c) Give all square-roots of 6.

   (d) How many rows of the multiplication table contain every element?

5. By definition a number has exactly 2 factors if and only if it is prime. When does it have exactly 4 factors?

6. A famous mathematician once noticed that the formula $f(n) = n^2 - n + 41$ yields primes for small values of $n$. For example, when $n = 1$ he calculated $f(1) = 41$, which is prime. When $n = 2$ he calculated $f(2) = 43$ and this is prime.

   (a) Test the formula for $n = 3$, 4, and 5. Are the results prime?

   (b) Does the formula always yield primes? Prove your answer.

Due: Thursday 5 October

Game of the Week. You and your classmates find yourself imprisoned by a sadistic dictator. He tells you that in the morning you will all be put in a room around a table, and each person will get a hat placed on their head, either red or blue. Each person will be able to see everyone else’s hat, but not their own. No communication will be possible inside the room. Every person will simultaneously be required to guess what color their hat is. If everyone guesses right, then all live, otherwise everyone dies. Fortunately, a benevolent mathematician persuades him to allow everyone to live if either everyone guesses right or if everyone guesses wrong. What should your class’ strategy be?