Math 4190 — Goddard — Spring18
Assignment 4

You may work in pairs, and submit one answer sheet for the pair.

1. Let $p$ be a prime number other than 2. Prove that $2^p$ cannot be written as the difference of squares (of integers).

2. List all factors of 945.

3. Define $D(n)$ as the number of factors of $n$. For example, the primes are the $n$ such that $D(n) = 2$.
   (a) Determine all $n$ such that $D(n) = 3$.
   (b) Determine all $n$ such that $D(n) = 4$.

4. Calculate the gcd of:
   (a) 91 and 287.
   (b) $12^{100}$ and $100^{12}$.

5. Let $a$ be a positive integer. Prove that $2a + 1$ and $4a^2 + 1$ are relatively prime.

6. Show that the gcd operation is associative. That is: $\gcd(a, \gcd(b, c)) = \gcd(\gcd(a, b), c)$.

7. Consider $\mathbb{Z}_{10}$.
   (a) List all elements of $\mathbb{Z}_{10}$.
   (b) What is the inverse of 3?
   (c) Give all square-roots of 6.
   (d) How many rows of the multiplication table contain every element?

Due: Friday 23 February

Game of the Week. You and your classmates find yourself imprisoned by a sadistic dictator. He tells you that in the morning you will all be put in a room around a table, and each person will get a hat placed on their head, either red or blue. Each person will be able to see everyone else’s hat, but not their own. No communication will be possible inside the room. Every person will simultaneously be required to guess what color their hat is. If everyone guesses right, then all live, otherwise everyone dies. Fortunately, a benevolent mathematician persuades him to allow everyone to live if either everyone guesses right or if everyone guesses wrong. What should your class’ strategy be?