1. Consider the following identity:

\[
\sum_{0 \leq i \leq n/2} \binom{n}{2i} = \sum_{0 \leq i < n/2} \binom{n}{2i + 1}.
\]

For example, when \( n = 3 \) it claims that \( \binom{3}{0} + \binom{3}{2} = \binom{3}{1} + \binom{3}{3} \); this is true since both sides equal 4.

(a) Verify this identity for \( n = 4 \) and \( n = 5 \).
(b) Deduce this identity from the Binomial Theorem (by plugging in suitable value of \( x \) and \( y \)).
(c) Give a combinatorial proof of the identity.

2. Consider the function \( f(x) = x^2 \).

(a) Give an example of domain and codomain such that the function \( f \) is onto but not 1–1.
(b) Give an example of domain and codomain such that the function \( f \) is 1–1 but not onto.
(c) Give an example of domain and codomain such that the function \( f \) is a bijection.

3. How many partitions are there of a 5-element set?

4. Let \( < \) be the “less-than” relation with universe the positive integers;
   let \( D \) be the relation with universe all sets, such that two sets are related if they are disjoint
   let \( \approx \) be the relation with universe all real numbers, such that \( x \) and \( y \) are related if \( |x - y| < 0.01 \).

Complete the following table (with “yes” and “no”):

<table>
<thead>
<tr>
<th></th>
<th>&lt;</th>
<th>( D )</th>
<th>( \approx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transitive</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

5. Show that the product of four consecutive integers is a multiple of 12.

6. Give a proof by contradiction that if \( x, y, z \) are integers such that \( xyz \leq 1000 \), then at least one of \( x, y, z \) is at most 10.

Due: Start of class: Friday 9 February