1 General MATLAB

We type commands into the Command Window. The semi-colon suppresses reporting the result. One can store the results of expression in variables (which are not pre-typed). MATLAB is case sensitive: A sequence of commands can be saved as a script. Comments are added by preceding with the percentage sign.

MATLAB has the expected arithmetic operators, constants and standard functions. Value \texttt{Inf} (standing for infinity) and \texttt{NaN} (standing for Not-a-Number).

Character vectors are enclosed in single quotes; strings are enclosed in double quotes (but most of the time interchangeable). Data can be obtained from the user with the \texttt{input} command. The command \texttt{format} adjusts the display; but a more general is \texttt{fprintf}, which takes a “format-string” followed by values.
Logical expressions are either `true` (1) or `false` (0). Double-equals is used for testing equality; `~` for not, `& &` for and, and `||` for (inclusive) or.

The simplest conditional statement uses an `if` to evaluate a logical expression and then execute some code only if the expression is true. Optionally one can provide an `else` that gives code to be executed when the condition is false. There is also `elseif`.

A `for` loop executes the body of the loop for every value of the index (or loop) variable. (Changing loop variable in the body has no permanent effect.) An infinite loop can be terminated by Ctrl-C.

A `while` loop executes the body as long as some condition remains true. (The condition is always checked: so the body might never be executed.) Often the loop is preceded by initialization of a loop variable. Loops can be nested.
A (row) **vector** is entered by enclosing values in square brackets, separated by spaces and optionally commas. Positions are numbered from 1 in MATLAB. The expression \( A(2) \) gives the second element, the expression \( 2:4 \) is the vector \([2 \ 3 \ 4]\) , and \( A(2:4) \) gives the sub-vector of \( A \) from position 2 to 4.

There are inbuilt vector functions e.g. `median` or `max`. One can apply an operator entry-wise on an array by using a dot before the operator. Functions designed for a single value, such as `sin`, are automatically applied entry-wise on a vector.

If \( A \) and \( B \) are vectors, the expression \( A==B \) yields a logical vector that is 1 where \( A \) and \( B \) agree, and 0 where \( A \) and \( B \) disagree. Similarly for other relation operators. For a logical vector, function `all` tests if all entries are true, and `any` tests if at least one entry is true.
4 Functions and 2D Arrays

We store a function in a function file of the same name. It works with its own local copy of variables that it is passed. A function can have multiple outputs, or even no output. It is specified by `function OUTPUT = FUNCTION_NAME ( INPUT )`

A function can be passed to another function by making its name a string. An anonymous function is defined using at-notation such as `mySquare = @(x) x^2;`

A list of strings can be stored in an array of `strings` (using double quotes). The `blanks` command creates a blank char-vector of specified length.

A matrix (or table) is a two-dimensional. There are inbuilt matrices such as `zeros` and `ones`. A matrix can also be specified using a semi-colon to indicate the end of each row. Individual entries are accessed by giving the row and column; for example `M(2,3)`. One can extract an entire row or column by using a colon. The `size` function returns the dimensions.
5 Numerical Programming

A Monte Carlo method uses computer simulation (with random choices for unknowns) to obtain an answer. MATLAB functions for generating random numbers include `rand` and `randi`. We discussed the “birds on a wire problem”.

To find the root of function $f$, the bisection method starts with an interval $[a, b]$ such that $f(a)$ and $f(b)$ have opposite signs. It repeatedly calculates the midpoint of the interval, and retains that half guaranteed to contain a root.

The secant method calculates a sequence of points, each an approximation to the root, by each stage joining the two previous values and seeing where that line-segment crosses the $x$-axis. A related idea is regula falsi. In Newton’s Method, rather than using a secant from the last two values, one uses the tangent at the previous value.

The area under a curve (numerical integration) can be approximated by breaking up the area into equal-sized strips. The midpoint rule estimates the area of a strip by using the function value at the middle; the trapezoidal rule by using the function values at the ends; and Simpson’s rule fits a parabola to the three points.
6 Graphics

The function \texttt{fplot} plots a curve when provided a function (handle). The function \texttt{plot} creates a plot joining a sequences of points: it is provided two vectors, one with the x-coordinates and one with the y-coordinates. There are many ways to enhance the plot.

A primitive way of producing graphics in MATLAB is to use the \texttt{rectangle} command. This produces a rectangle whose lower-left corner is at a specified point and whose size is specified. One can also specify the edge-color, the fill-color, and other features. We used this to draw flags by building up a picture using rectangles as square pixels.

Pascal’s Triangle has a first row that is a single 1, and every entry in subsequent rows is the sum of the two entries above it. If one colors the numbers by their remainder when divided by some value, one gets a fractal-like picture. Another fractals include Sierpinski’s carpet, Sierpinski’s gasket, and the Koch snowflake. These can be drawn using recursion. The \texttt{fill} command colors in a polygon.
7 Games

A combinatorial game such as tictactoe is where two players alternate but there is no randomness or hidden information. The game can be represented by a game tree where for each node, the children are the next positions. Using this tree, it can be shown that for a game that cannot end in a tie or go on forever, either there is a strategy that guarantees the first player a win, or there is a strategy that guarantees the second player a win.

A simultaneous game such as rock-paper-scissors is where players independently choose an option, and the result is determined by the options chosen. A two-person game is zero-sum if what one player loses the other player gains. A simultaneous two-person zero-sum game can be represented by a matrix: the two players are RowPlayer, who chooses a row, and ColumnPlayer, who chooses a column; and the table gives RowPlayer’s gain/ColumnPlayer’s loss. The optimal strategy for each player is to randomly choose each option with a certain probability. For $2 \times 2$ games, we can solve by hand by looking for one option dominating another, and failing that, choosing the proportions to make the opponent’s actions equal.

In Mastermind, the Maker chooses a secret and the Breaker asks questions about the secret in the form of guesses; these are scored on their closeness to the secret. We looked at MATLAB code to do the scoring, code for a Maker player, and code for a Breaker player that each time picks a guess at random from the remaining possible secrets.
Adversarial search entails trying to determine a hidden object using a series of questions (such as in the game 20 Questions). The search strategy can be represented by a decision tree, which is laid out like a family tree with the first question at the top and the children being the follow-up questions. The tree continues until the object is identified.

A natural idea is to start with the question that gives the “most” information. If the question creates sets with proportions (or probabilities) $p_1, p_2, \ldots, p_m$, then the Shannon entropy of the question is given by $I = -\sum_{i=1}^{m} p_i \log_2 p_i$.

There are many ways to attempt to rank sports teams, websites, etc. Massey’s original idea was to assign each team a numeric rating such that in an ideal world the net result of any game is the difference between the ratings of the teams. For a practical situation, we calculate the numerical ratings that have the least square error.

In the Elo system, ratings are continually updated. A player is rewarded if their result is better than expected and their opponent penalized by the same amount. The adjustment is based on a distribution that is given by a logistic curve.

In the Markov method one takes a random walk through the network and the rating of a site is the steady-state average time spent at that vertex. In PageRank, each site starts with one dollar and then distributes its money equally to all those that point to it; this is then iterated until convergence. PageRank and Markov Methods are equivalent.