H1.1 Adversarial Search

Consider the problem of trying to determine a hidden object using a series of questions, such as in the game 20 Questions. Specifically, there is a finite universe $X$ and we get to ask a sequence of questions to identify a particular element of $X$. Each question splits the current universe into subsets depending on the possible answers; for example, a yes–no question splits the current universe into two subsets. We can depict the overall strategy by a decision tree, which is laid out like a family tree with the first question at the top and the children being the follow-up questions. The tree continues until each subset has only one element.

![Decision Tree Diagram](image)

H1.2 Finding the Optimal Decision Tree

What is the best strategy? Well, if the universe is the set of animals, you could start by asking whether it is a mammal with black and white stripes: this is great if the answer is yes, but useless if the answer is no. It is better to ask a question where either answer narrows the candidates down a lot. In other words, ask a question where either answer provides a lot of information.

A natural idea then is the following strategy often used in artificial intelligence. It is called induction in the logic sense, meaning reasoning from examples (in contrast to deduction).

Tree Induction

*Find best question*
Partition universe as per question
Repeat for each piece

If you are only allowed questions that have two possible answers, then it is natural to choose the question that splits the universe as evenly as possible. But how do we determine the best question in general?

### H1.3 Entropy

One way for the computer in general is to choose the guess that yields the most information. The mathematical definition of information is called entropy. If the question creates sets with proportions (or probabilities) \( p_1, p_2, \ldots, p_m \), then the Shannon entropy or information of the question is given by

\[
I = -\sum_{i=1}^{m} p_i \log_2 p_i.
\]

The bigger \( I \) is, the more information answering the question provides. For example, if we have a question that has the same answer for everything, meaning \( m = 1 \) and \( p_1 = 1 \), then \( I = 0 \). If we have a question that splits the universe into two equal parts, meaning \( m = 2 \) and \( p_1 = p_2 = \frac{1}{2} \), then \( I = 1 \).

**Example.** Often a question is based on an attribute or property of the elements of the universe. Consider for instance the classification of animals.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Fur</th>
<th>Color</th>
<th>Small</th>
<th>Herbivore</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kodiak</td>
<td>yes</td>
<td>brown</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Lion</td>
<td>yes</td>
<td>yellow</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Aardvark</td>
<td>yes</td>
<td>brown</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Canary</td>
<td>no</td>
<td>yellow</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Salmon</td>
<td>no</td>
<td>pink</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

Attribute: Herbivore: \( I = -\frac{4}{5} \log_2 \frac{4}{5} - \frac{1}{5} \log_2 \frac{1}{5} \approx 0.72 \)

Attribute: Color: \( I = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{2}{5} \log_2 \frac{2}{5} - \frac{1}{5} \log_\frac{1}{5} \approx 1.52 \).

Of course, the question that reveals the most information is “What animal is it?”