(a) List the three row operations.

*interchange two rows; scale a row; add multiple of one row to another row*

(b) What’s the maximum number of nonzero entries in a $3 \times 3$ matrix in reduced row echelon form?

*Four. In the case that there are pivots in first two columns only.*

(c) In $\mathbb{R}^2$, give a set of three vectors that:

- (a) span $\mathbb{R}^2$
  
  *E.g. $(1, 0), (0, 1)$ and any other vector*

- (b) don’t span $\mathbb{R}^2$
  
  *E.g. $(1, 0), (2, 0), (3, 0)$*

- (c) are linearly independent
  
  *Do not exist*

- (d) are not linearly independent
  
  *Any three*
(d) Assume $T$ is a linear transformation with $T((1,0)) = (3,2)$ and $T((1,1)) = (-1,4)$. What is the associated matrix transformation?

*The columns of the matrix are the images of $(1,0)$ and $(0,1)$. Note that $T(0,1) = T(1,1) - T(1,0)$. So matrix is $\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$.*

(e) If $A$ is $2 \times 2$, $B$ is $2 \times 5$, and $C$ is $5 \times 1$, which of the following products are defined? $A^3$, $BC$, $CB$, $BA^T$

$A^3$ and $BC$ are defined

(f) Give in each case a $2 \times 2$ matrix that is:

(a) equal to its transpose

*E.g. The identity or the all-zero matrix*

(b) equal to its inverse

*E.g. The identity*

(c) lower triangular but not invertible

*E.g. The all-zero matrix*