1. Consider question 4 of previous assignment. For the cases where the set is a subspace, give the dimension and a basis.

2. Give example(s) that shows that the intersection of two $d$-dimensional spaces can have dimension anywhere from 0 to $d$.

3. Consider the following triplets of vectors in $\mathbb{R}^3$. In each case state whether the triplet is a basis, spans $\mathbb{R}^3$ and/or is linearly independent.
   
   (a) $(0, 0, 0), (1, 1, 1), (2, 2, 2)$
   (b) $(1, 1, 1), (1, 1, 0), (1, 0, 0)$
   (c) $(0, 1, 2), (2, 0, 1), (1, 2, 0)$

4. Consider the matrix

   \[
   G = \begin{bmatrix}
   1 & 2 & 0 & -3 \\
   2 & 7 & 1 & 7 \\
   -2 & -1 & 1 & 19
   \end{bmatrix}
   \]

   (a) Give the dimension and a basis of the column space of $G$.
   (b) Give the dimension and a basis of the null space of $G$.
   (c) Give the dimension and a basis of the row space of $G$.

5. Say $S$ and $T$ are $9 \times 9$ matrices of rank 5. Consider the matrix product $ST$.
   
   (a) Give an example of a $9 \times 9$ matrix of rank 5.
   (b) Prove that the product $ST$ always has rank at most 5. (Hint: the columns of $ST$ are linear combinations.)
   (c) Prove that the product $ST$ is never zero. (Hint: null spaces and column spaces)

Due: Start of class, Friday 19 October