[Assignment 7]

(If you prefer, you may work in a pair and submit one solution for the pair.)

1. Prove that a linear transform always maps \( \mathbf{0} \) to \( \mathbf{0} \).

2. Consider Exercise 3 of the previous assignment. For the cases where the set is a vector space, give the dimension and a basis.

3. Suppose that \( S = \{ \mathbf{x}, \mathbf{y}, \mathbf{z} \} \) is a linearly dependent set. Prove that every vector \( \mathbf{v} \) in the span of the set \( S \) can be expressed as a linear combination in more than one way.

4. Let \( L \) be the set of all linear transforms from \( \mathbb{R}^3 \) to \( \mathbb{R}^2 \).
   (a) Verify that \( L \) is a vector space.
   (b) Determine the dimension of \( L \) and give a basis for \( L \).

5. Consider the matrix
   \[
   F = \begin{bmatrix}
   2 & -1 & 0 & -3 \\
   12 & -6 & 1 & 7 \\
   0 & 0 & 0 & 0
   \end{bmatrix}
   \]
   (a) Give the dimension and a basis of the column space of \( F \).
   (b) Give the dimension and a basis of the null space of \( F \).
   (c) Give the dimension and a basis of the row space of \( F \).

Due: Friday Nov 1