1. Calculate the determinants of the following matrices using cofactors.

\[ D = \begin{bmatrix} 1 & -1 & 3 \\ 3 & 4 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad L = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 2 & 0 \\ 4 & 1 & 5 \end{bmatrix} \]

2. Calculate the determinants of the matrices in the previous question using row reduction.

3. A **permutation matrix** is an \( n \times n \) matrix with 0's everywhere except for exactly one 1 in every row and column. (For example the identity matrix is a permutation matrix.)

   (a) Show that every permutation matrix has determinant 1 or \(-1\).
   
   (b) Show that the transpose of a permutation matrix is its inverse.
   
   (c) Show that the product of two permutation matrices is a permutation matrix.

4. Suppose \( A \) is a \( 3 \times 3 \) matrix such that \( \det A = 3 \). Give the determinant of:

   (a) \( A^T \)
   
   (b) \((A^2)^{-1}\)
   
   (c) The matrix that results if one takes \( A \) and **replaces** the 2nd row by the sum of the 1st and 3rd rows.
   
   (d) The matrix that results if one takes \( A \) and **increases** the 2nd row by the sum of the 1st and 3rd rows.
   
   (e) \( A + A \).
   
   (f) \( A + A^T \).

5. (a) Prove that if two rows of a matrix are identical then its determinant is 0.

   (b) Assume \( A \) is a square matrix with first row \( R \). Let \( B \) be the square matrix obtained from \( A \) by replacing the first row by \( S \). Let \( C \) be the square matrix obtained from \( A \) by replacing the first row by \( R + S \). Prove that \( \det A + \det B = \det C \).

   (c) Use the above results to prove that the “replacement” elementary row operation does not change the determinant.

Due: Mon October 7