1. Solve the following systems.

\[
\begin{align*}
    s + t &= 5 \\
    2s + t &= 1 \\
    x - y + z &= 8 \\
    2x - y &= 4 \\
    3y &= -6
\end{align*}
\]

2. For what value(s) of \( \beta \) is the system \( x + \beta y = 1 \) and \( x - y = \beta \) consistent?

3. Consider this matrix

\[
B = \begin{bmatrix}
    0 & 1 & 0 & 0 & 0 & -3 \\
    0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & -5 \\
    0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(a) Is matrix \( B \) in echelon form? Explain.

(b) Is matrix \( B \) in reduced row echelon form? Explain.

4. Describe geometrically the solution to the linear equation \( x + y + z = 1 \).

5. Consider \( n \) numbers \( x_1, x_2, \ldots, x_n \) laid out on a circle and some value \( \alpha \). Consider the requirement that every number equals \( \alpha \) times the sum of its two neighbors. For example, if \( \alpha \) were zero, this would force all the numbers to be zero.

(a) Show that, no matter what \( \alpha \) is, the system has a solution.

(b) Show that if \( \alpha = \frac{1}{2} \), then the system has a nontrivial solution.

(c) Show that if \( \alpha = -\frac{1}{2} \), then there is a nontrivial solution if and only if \( n \) is even.

Due: Wednesday August 28