Warmup 4: Context-free Languages and TMs

1. State whether each of the following is true or false. No justification required.

   (a) There exists a language accepted by a TM but by no PDA.
      \[ \text{TRUE} \quad (e.g. \ 0^n1^n2^n) \]

   (b) There exists a language accepted by a nondeterministic FA but by no deterministic PDA.
      \[ \text{FALSE} \quad \text{(convert NFA to DFA)} \]

   (c) The context-free language are closed under the three Kleene operations.
      \[ \text{TRUE} \]

   (d) If a language \( L \) is accepted by a deterministic PDA, then it is guaranteed that the complement of \( L \) is accepted by some deterministic PDA.
      \[ \text{TRUE} \]

2. Let \( X \) be the set of all binary palindromes with equal 0's and 1's. Consider a proof using the Pumping Lemma that \( X \) is not context-free, and assume \( k \) is the constant of the Pumping Lemma. For each of the following say, with a brief justification, whether it would be suitable for use in such a proof.

   (a) \( 0^{2k}1^{2k} \)
      \[ \text{No. Not in X} \]

   (b) \( 0^{2021}1^{4042}0^{2021} \)
      \[ \text{No. Not guaranteed to be long enough.} \]

   (c) \( (01)^k(10)^k \)
      \[ \text{No. It is pumpable: For uvwxy, let u be last 01 of first half and x be first 10 of second half.} \]
3. Consider the following TM with input alphabet \{0, 1\}:

(a) Give two strings of length 3 accepted by the PDA.
(b) Give two strings of length 3 NOT accepted by the PDA.
(c) Describe in succinct-ish English the language of this PDA. Be precise.

All binary strings where number of 0's and number of 1's differ by exactly one.