Supplemental Questions on: Regular Languages

B1: (a) For the leftmost NFA below, use the subset construction to give a DFA that is equivalent to it.

(b) For the rightmost NFA below, use the GFA approach to construct an RE that is equivalent to it.

\[ \text{Diagram of the leftmost NFA} \]

\[ \text{Diagram of the rightmost NFA} \]

B2: Suppose language \( L \) is accepted by FA \( M \). Define \( L^E \) as the set of all strings in \( L \) that have even length. Explain how to construct an FA that accepts \( L^E \).

B3: Apply the subset construction to produce a DFA that accepts the same language as the following NFA.

\[ \text{Diagram of the NFA} \]

B4: (a) Given language \( L \), the language \( L^F \) is defined as the set of all final segments of strings in \( L \). For example, if \( L = \{ \text{DO}, \text{GOOD} \} \), then \( L^F = \{ \text{DO}, 0, \varepsilon, \text{GOOD}, \text{OOD}, \text{OD}, \text{D} \} \). Provide an algorithm that converts an FA for language \( L \) to one for language \( L^F \).

(b) For a language \( L \), let \( L^o \) be the set of all strings obtained from all strings of \( L \) by omitting one symbol from the string. For example, if \( L \) contains \( \text{dog} \), then \( L^o \) contains \( \text{og}, \text{dg}, \text{and do} \). Show how to build an FA for \( L^o \) given an FA for \( L \).

(c) Given a string \( z \), the sampling of \( z \) is the string consisting of every alternate character in \( z \), starting with the first. For example, the sampling of \( \text{Goddard} \) is \( \text{Gdad} \). Describe an algorithm that converts a DFA for language \( L \) to an FA for the set of all samplings of strings in \( L \).

B5: (a) Give an example of nonregular languages \( L_1 \) and \( L_2 \) such that \( L_1 \cup L_2 \) is regular.

(b) Give an example of a regular language such that any DFA accepting it requires at least 100 states.
B6: Explain how to tell whether a language is $\Sigma^*$ or not:
  (a) Given a DFA for the language.
  (b) Given an RE for the language.

B7: Give a biggest possible set of pairwise distinguishable strings with respect to:
  (a) the set of all binary strings containing 110 as a substring.
  (b) the set of all binary strings not containing 000 as a substring.
  (c) the set of all binary strings starting with 011.

B8: Use the Pumping Lemma to show that the following languages is not regular:
   $\{0^n1^n0^n : n \geq 10 \}$.

B9: Consider the language $B = \{0^n1^n : n \geq 0 \}$ and let $C$ be a subset of $B$. Use the Pumping Lemma to show that if $C$ is infinite, then $C$ is not regular.

B10: Let $L$ be the language corresponding to the RE $(0+1)^*(00+11)(0+1)^*$. Find a set of maximum size of strings that are pairwise distinguishable with respect to $L$. 