Let $A$ be an alphabet, and let $f$ be a function that maps each symbol in $A$ to some nonempty string. Given a string $w$ in $A^*$, we define the string $w^f$ as replacing every symbol in $w$ by its corresponding $f$ value. And we define for language $L$ the language $L^f$ as the set of all $w^f$ for $w$ in $L$.

(a) Show that if $L$ is r.e. then so is $L^f$.

Here is procedure for $L^f$.
Consider the input string $x$.
Generate all possible strings $w$ of length at most $|x|$ from alphabet $A$.
For each such $w$ calculate $w^f$.
For all the $w$ such that $w^f = x$:
run the machine for $L$ on the string $w$, doing all of this in parallel.
If any of the machines accept their $w$, stop and accept.
If all the machines terminate and reject their $w$, reject.

(b) Show (by means of an example) that $L^f$ can be regular even if $L$ is not.

Consider your favorite non-regular binary language $L$ that contains a string of all possible lengths.
E.g. binary strings with unequal 0’s and 1’s.
Then let $f$ map both 0 and 1 to the symbol #.
Then $L^f$ is all strings of #, which is clearly regular.