Warmup 2: Regular Languages and CFGs

[ about 3/4 length of actual test ]

1. Given a string \( x \), an expansion of \( x \) is any string obtained by repeating some of the letters some number of times. For example, each of CCAATT, CAT and CCCCCCAAT are expansions of CAT. Given a language \( L \), the expansion of \( L \) is all possible expansions of strings in \( L \). Show that the regular languages are closed under expansion.

   e.g. transform \( RE \) by replacing each char \( x \) by \( xx^* \)
   e.g. \( (0+11)*0 \) \( \Rightarrow (00^*+11^*11^*)00^* \)

2. For each language, give 3 strings that are pairwise distinguishable with respect to that language:
   (a) The set of all binary strings whose first and last bit are the same
      e.g. \( 11, 1001 \)  
      \[ \text{[first and last bit matter]} \]
   (b) The set of all binary strings that contain 101 as substring
      e.g. \( \varepsilon, 110 \)  
      \[ \text{[progress on containing 101]} \]
   (c) The set of all binary strings of odd length.
      \[ \text{only can find 2 distinguishable} \]

3. Give a CFG for the set of all even-length palindromes from alphabet \( \{a, b\} \) that contain \( abba \) as a substring.

   \[ P \rightarrow aPa | bPb | abbaQ | abba \]
   \[ Q \rightarrow aQa | bQb \varepsilon \]  
   \[ \text{[Q does all even-length palindromes]} \]

4. Consider the following CFG with start variable \( S \):

   \[
   S \rightarrow 0T0 | 1T1 | 0T1 | 1TO | \varepsilon \\
   T \rightarrow 0S | 1S | \varepsilon 
   \]

   (a) Give a derivation tree for the string 01010

   (b) Describe in English the language of this grammar.

   \[ \text{all binary strings} \]
   \[ \text{except single 0 or 1} \]